Solve the system of equations **SCORE: _____ / 12 PTS**

$$-2x-11y+2z=-13$$
$$3x+12y-9z=27$$
$$4x+21y-2z=21$$

NOTES: **You must use the pivot method shown in lecture, and you must state all elementary row operations performed.**
You must produce a matrix in reduced row echelon form (RREF) (if the system has a solution) or row echelon form (REF) (if the system has no solution).
You must check that your final answer is correct (if the system has a solution).

$$\left[\begin{array}{cccc} -2 & -11 & 2 & -13 \\ 3 & 12 & -9 & 27 \\ 4 & 21 & -2 & 21 \end{array}\right] R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{cccc} 3 & 12 & -9 & 27 \\ -2 & -11 & 2 & -13 \\ 4 & 21 & -2 & 21 \end{array}\right] \tfrac{1}{3}R_1$$

$$\left[\begin{array}{cccc} 1 & 4 & -3 & 9 \\ -2 & -11 & 2 & -13 \\ 4 & 21 & -2 & 21 \end{array}\right] \begin{array}{l} \\ R_2+(2)R_1 \\ R_3+(-4)R_1 \end{array}$$

$$\left[\begin{array}{cccc} 1 & 4 & -3 & 9 \\ 0 & -3 & -4 & 5 \\ 0 & 5 & 10 & -15 \end{array}\right] R_2 \leftrightarrow R_3$$

$$\left[\begin{array}{cccc} 1 & 4 & -3 & 9 \\ 0 & 5 & 10 & -15 \\ 0 & -3 & -4 & 5 \end{array}\right] \tfrac{1}{5}R_2$$

$$\left[\begin{array}{cccc} 1 & 4 & -3 & 9 \\ 0 & 1 & 2 & -3 \\ 0 & -3 & -4 & 5 \end{array}\right] R_3+(3)R_2$$

$$\left[\begin{array}{cccc} 1 & 4 & -3 & 9 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 2 & -4 \end{array}\right] \tfrac{1}{2}R_3$$

$$\left[\begin{array}{cccc} 1 & 4 & -3 & 9 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 1 & -2 \end{array}\right] \begin{array}{l} R_1+(3)R_3 \\ R_2+(-2)R_3 \\ \\ \end{array}$$

$$\left[\begin{array}{cccc} 1 & 4 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right] R_1+(-4)R_2$$

$$\left[\begin{array}{cccc} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$(x, y, z) = (-1, 1, -2)$$

$$-2(-1)-11(1)+2(-2)=2-11-4=-13$$
$$3(-1)+12(1)-9(-2)=-3+12+18=27$$
$$4(-1)+21(1)-2(-2)=-4+21+4=21$$

Solve the following systems of equations **SCORE: _____ / 18 PTS**

[a]
$$\begin{aligned} 3x+4y-11z&=5\\ 2x+y-4z&=5\\ -x-2y+5z&=-1 \end{aligned}$$

[b]
$$\begin{aligned} 3x+5y-9z&=2\\ 2x-3y+13z&=-1\\ -x+2y-8z&=3 \end{aligned}$$

NOTES: **You must use the pivot method shown in lecture, and you must state all elementary row operations performed. You must produce a matrix in reduced row echelon form (RREF) (if the system has a solution) or row echelon form (REF) (if the system has no solution).**

$$\left[\begin{array}{cccc} 3 & 4 & -11 & 5\\ 2 & 1 & -4 & 5\\ -1 & -2 & 5 & -1 \end{array}\right] R_1 \leftrightarrow R_3$$

$$\left[\begin{array}{cccc} -1 & -2 & 5 & -1\\ 2 & 1 & -4 & 5\\ 3 & 4 & -11 & 5 \end{array}\right] -R_1$$

$$\left[\begin{array}{cccc} 1 & 2 & -5 & 1\\ 2 & 1 & -4 & 5\\ 3 & 4 & -11 & 5 \end{array}\right] \begin{array}{l} \\ R_2+(-2)R_1\\ R_3+(-3)R_1 \end{array}$$

$$\left[\begin{array}{cccc} 1 & 2 & -5 & 1\\ 0 & -3 & 6 & 3\\ 0 & -2 & 4 & 2 \end{array}\right] -\tfrac{1}{3}R_2$$

$$\left[\begin{array}{cccc} 1 & 2 & -5 & 1\\ 0 & 1 & -2 & -1\\ 0 & -2 & 4 & 2 \end{array}\right] R_3+(2)R_2$$

$$\left[\begin{array}{cccc} 1 & 2 & -5 & 1\\ 0 & 1 & -2 & -1\\ 0 & 0 & 0 & 0 \end{array}\right] R_1+(-2)R_2$$

$$\left[\begin{array}{cccc} 1 & 0 & -1 & 3\\ 0 & 1 & -2 & -1\\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{aligned} x-z&=3 \\ y-2z&=-1 \end{aligned} \quad \Rightarrow \quad \begin{aligned} x&=z+3\\ y&=2z-1 \end{aligned}$$

$$(x,\, y,\, z)=(z+3,\, 2z-1,\, z)$$

$$\left[\begin{array}{cccc} 3 & 5 & -9 & 2\\ 2 & -3 & 13 & -1\\ -1 & 2 & -8 & 3 \end{array}\right] R_1 \leftrightarrow R_3$$

$$\left[\begin{array}{cccc} -1 & 2 & -8 & 3\\ 2 & -3 & 13 & -1\\ 3 & 5 & -9 & 2 \end{array}\right] -R_1$$

$$\left[\begin{array}{cccc} 1 & -2 & 8 & -3\\ 2 & -3 & 13 & -1\\ 3 & 5 & -9 & 2 \end{array}\right] \begin{array}{l} \\ R_2+(-2)R_1\\ R_3+(-3)R_1 \end{array}$$

$$\left[\begin{array}{cccc} 1 & -2 & 8 & -3\\ 0 & 1 & -3 & 5\\ 0 & 11 & -33 & 11 \end{array}\right] R_3+(-11)R_2$$

$$\left[\begin{array}{cccc} 1 & -2 & 8 & -3\\ 0 & 1 & -3 & 5\\ 0 & 0 & 0 & -44 \end{array}\right]$$

NO SOLUTION